The Development of MVAR and TVAR

My thesis would not have been possible had not Jim given me the book by Sir Michael James Lighthill. Lighthill has become one of my heroes. He was a brilliant applied mathematician. He solved the problem of jet silencing sufficient so that airports could be close to cities. It doesn't work above Mach 1, so military aircraft are often a lot louder. He held the Newton's Chair before Stephen Hawking. He said of Hawking's cosmology, "It's not quite science. It's more like creation myth." Lighthill's book for me is a classic: *Introduction to Fourier Analysis and Generalized Functions*. I have turned to the chart on page 43 probably a hundred times, and that's probably small compared to the number of times I've used it. It is very, very powerful. It is his derived transform relationships that allows us to go from the time-domain to the frequency-domain so elegantly -- making what we often call a super-fast Fourier transform.

There is a simple and elegant relationship between the time-domain and the frequency-domain as obtained from page 43 of Lighthill's book. If AVAR is proportional to τ^{mu} and the spectral density is proportional to f^{alpha}, then we may derive that alpha = - mu - 1, and we have this super-fast Fourier-transform relationship from -2 < mu < 3. At mu = -2, we have the ambiguity problem for AVAR; there, alpha is greater than or equal to 1. We solved this ambiguity problem 16 years later with the Modified Allan variance.

When we discovered this ambiguity problem back in 1965, Bob Vessot, who was the reader for my thesis for the 1966 Proceedings saw that I had a mistake in my master's thesis as published at CU. Jim and I went back to Lighthill's book and sorted this out. Bob was especially interested in this problem, because of the excellent short-term stability of his hydrogen masers having white-noise PM modulation. He knew, and we knew that this kind of noise was bandwidth dependent, even though we had the ambiguity problem for AVAR at mu = -2. One could differentiate between white-noise PM and flicker-noise PM by modulating the bandwidth, but that is a nuisance. "Equation 34" of my thesis showed the τ dependence as well as the bandwidth dependence, but we didn't know how to do it in software back then. It wasn't until 1981 that we discovered how to do software bandwidth modulation and we found the elegant solution with the Modified Allan variance.

In a typical ADEV plot (sigma-tau diagram), we often see a bath-tub like curve resulting from the different power-law spectral density processes used in modeling atomic clocks. For the white noise FM coming from a passive atomic clock, we see the $\tau^{-1/2}$ slope. The next segment is the zero-slope flicker-noise FM region of tau values, sometimes called the flicker floor. The third segment is the model for random-walk FM -- giving rise to a $\tau^{+1/2}$ slope. The last segment is what one observes if there is frequency drift in the data. Then you get a τ^{+1} slope. However, if we have flicker-noise PM or white-noise PM on an ADEV plot, we can't distinguish them, which illustrates the ambiguity problem at mu = -2.

We made progress on understanding how to distinguish flicker-noise PM and white-noise PM with the development of MDEV in 1981. With an MDEV plot, one can distinguish between white-noise PM and flicker-noise PM, where the slopes go from $\tau^{-3/2}$ to τ^{-1}, respectively. This work was triggered when Dr. James J. Snyder came out to JILA from Gaithersburg, Maryland to do studies on short-term stability on lasers. He invited Jim Barnes and me to go over to JILA and see his results. He was able to get a $\tau^{-3/2}$ slope on a log-log sigma-tau diagram for the white-phase noise by averaging the phase.

So I was really excited when I saw his results; we had lived with this ambiguity problem for 16 years. A little side story. I had been invited to give a talk in New Delhi to an international conference for Third World countries on time and frequency transfer techniques, and on the flight from Denver to New Delhi, I was using my little programmable HP-65 calculator, to investigate these concepts that Dr. Snyder had shown us.

When I got there, I was higher than a kite. I sent a message to Jim Barnes that we needed to share these results at the next Frequency Control Symposium.

This was a major breakthrough in variance analysis which we were able to share in our paper in the 1981 Frequency Control Symposium. I remember well that we were in New Delhi in February 1981, as I was able to take my wife and celebrate our wedding anniversary in Guam on our way home on the 20th. I had found a $50 coupon to buy a second ticket for her from TWA. The UN paid for mine. After celebrating in Guam, we flew that evening to Hawaii, and it was again the 20th of February having crossed the International Date Line. So we got to celebrate that anniversary twice. Who but a "time-nut" would make that happen!

So this "breakthrough" not only helped the time and frequency community, it was picked up by the international navigation community as well. In 2015, I was invited to St. Petersburg, Russia to give a paper, give a plenary talk, and chair a panel around this subject at an international navigation conference. I was impressed with what they have done with the MVAR metric.

MVAR is most useful for the short-term stability of two-way satellite time and frequency transfer, for Telephone ACTS reciprocity using a 300 baud modem, and for GPS common-view. It was fascinating to observe that a 300 baud modem in reciprocity ACTS mode over a short distance could reach frequency instabilities of 3×10^{-11}.

In the late 1980s, the telecom community seeing what we had done for both time and frequency and for the navigation community, asked me if I would help them come up with a metric for telecom.

They gave us a bunch of their data, and Marc Weiss and I analyzed it extensively and came up with TVAR as a desirable metric for them. TVAR is proportional to MVAR with a $1/3\ \tau^2$ multiplier. The 3 in the denominator normalizes TVAR to be equal to the classical variance for white-noise PM -- the ideal time measurement noise. TVAR caught on with the national telecom community and then soon with the international community. The success of TVAR with the international telecom community won my wife and me a free trip to Edinburgh, Scotland in 2011 to receive the "Time Lord" award. So, they have found this metric very useful for network analysis. We are very happy to help them as well with their metrology problems, and while in Scotland we were able to visit the area where the Allan name originated.

Optimum Prediction:

These time-domain techniques not only work for clocks, but in many other areas of metrology -- allowing one to do optimum prediction, estimation, and smoothing, as well estimation of the effects coming from the systematics present, which are always there.

One can give examples of how to apply these statistics to do optimum prediction in the case of noises with even power-law spectral densities (white-noise PM, white-noise FM, which is the same as random-walk PM, and random-run FM). The optimum predictors for the flicker-noise process are more complicated but can be done using inverted Box and Jenkins ARIMA filtering techniques. There are also some simple and useful predictors for these flicker-noise processes, which we have shared during the NBS/NIST seminars.

As a rule of thumb, one can write the time-prediction error of a clock as τ_p times ADEV(τ_p), where τ_p is the time prediction interval in question. This prediction-error calculation is extremely useful in time prediction for GPS, since the GPS satellites have to predict time forward from one tracking and upload station to the next, which can be several hours long.

I have fun examples of these concepts with quartz-crystal oscillator clocks. In the first example we used computer simulation to model flicker-noise FM and the systematics in high-quality quartz oscillators; back then we only had an analog computer. Then we built a device that would automatically remove the systematics in the presence of flicker-noise FM. (Low Information Rate Time Control Unit, LIRTCU), and observed that the output behaved much like an atomic clock. It was used for a time to generate official time for NBS. https://tf.nist.gov/general/pdf/187.pdf AN ULTRA-PRECISE TIME SYNCHRONIZATION SYSTEM DESIGNED BY COMPUTER SIMULATION https://tf.nist.gov/general/pdf/180.pdf

In another example, I purposely bought the cheapest stopwatches I could find at Target -- $6 each with tax! I knew the 5-cent quartz-crystal oscillator used had excellent performance -- because the robotic technology has come a long way in that regard. Professor Neil Ashby, who did the relativity for GPS, helped me with this experiment. This experiment is one of the best examples I have seen in a tutorial sense of how to sort out the measurement variances (different kinds of noise), the individual clock variances, and the effects of the environment and measurement noise. We were able to show that these 5-cent crystal clocks have a flicker floor of 3 e-8; this only amounts to a time-dispersion error of about 0.3 seconds for the 140 days of the experiment, while the systematics were about 400 times larger. In other words, removing the systematics makes an enormous difference.